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Hot-Wire Measurements of the Heat Transfer in a Partially Magnetized Potassium Plasma[1]

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A hot-wire device designed as a gas-loaded heat pipe has been used to study the axial magnetic field influence on the heat transfer in a potassium vapor plasma at low vapor pressures $p_K = 400$ to 1200 Pa, temperatures T_w of the tungsten filament in the range 2000 to 2800 K, and magnetic field intensity $B = 0.182$ to 0.364 T. As a result of the applied magnetic field B, the measured thermal flux Q decreases. The separation between the heat fluxes transferred by radiation Q_r, by atoms Q_a, and by electrons Q_e can be accomplished analyzing the decrement ΔQ. A procedure to analyze the measured data in the presence and in the absence of the magnetic field in order to define the different mechanisms of the heat transfer has been introduced. The electron thermal conductivity can be obtained in the Frost approximation by means of the well-known transport phenomena theory using available electron–potassium atom cross sections in the range of electron energies $\varepsilon = 0.06$ to 2 eV.

KEY WORDS: electron thermal conductivity; heat transfer; hot-wire measurements; partially magnetized plasma; potassium vapors.

1. INTRODUCTION

The purpose of this work is to present experimental results on the magnetic field influence on heat transport in a partially magnetized potassium plasma. The measured quantities are analyzed using different available momentum transfer electron–potassium atom cross sections in the calculations.

The interest in the transport properties of the alkali metal vapors is significant [1, 2]. We have already reported results on the potassium vapor atom [3, 4] and electron thermal conductivity measured by means of a

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hot-wire device operating as a gas loaded heat-pipe. The advantages of this modification of the hot-wire method are obvious when it is applied to alkali metal vapors. In the traditional version of the hot-wire method, the vapor pressure calculated at the measured temperature of the coldest point in the experimental device involves a large uncertainty in the obtained thermal conductivity and makes the results practically not reproducible. These problems are completely precluded in the experimental device operating as a heat-pipe, by fixing the pressure of the external inert gas (Ar), we thereby define the potassium vapor pressure.

2. MEASUREMENTS

2.1. Experimental Device

The experimental device and the measurement procedure have been reported elsewhere [3, 4]. In short, the gauge was a Pyrex tube with an inner diameter of $2R = 10^{-2}$ m. A coaxial tungsten filament (diameter of $2r = 1 \times 10^{-4}$ m, length of 0.18 m) was heated by a stabilized DC current up to 2800 K, and the voltage was measured with a relative error less than 10^{-5}. The system was operating in a heat-pipe regime [4], the external gas being argon at a given pressure p_{Ar}. At the lower part of the gauge, the potassium was evaporated. The vapors condensed in the cold part outside the device and flowed down. With increasing glass tube heating, the vapors displaced the argon gas until a pure potassium atmosphere with the same pressure $p_{\mathrm{K}} = p_{\mathrm{Ar}}$ was obtained. In a gas loaded heat-pipe regime the heat losses of the wire were found to be constant over a large range of evaporation rates. The furnace containing the gauge was surrounded by a system of coaxial Helmholtz coils producing a uniform (within 1%) axial magnetic field B up to 0.36 T.

2.2. Energy Balance of the Hot Wire

Here we consider the tungsten wire temperatures $T_{\mathrm{w}} > 1700$ K when a plasma is generated near the filament surrounded by a gas with a low ionization potential such as potassium vapors. Then the total heat flux from the filament in the hot-wire device is

$$Q = UI = Q_{\mathrm{r}}(T_{\mathrm{w}}) + Q_{\mathrm{a}}(T_{\mathrm{a}}) + Q_{\mathrm{e}}(T_{\mathrm{e}}) \tag{1}$$

where U and I are the filament heating DC voltage and current, respectively. The radiation (Q_{r}), atom (Q_{a}), and electron (Q_{e}) conductivity heat fluxes are

$$Q_r = 2\pi r L \gamma \varepsilon_\nu (T_w^4 - T_R^4) \tag{2}$$

$$Q_a = A \int_{T_R}^{T_a} \lambda_a \, dT \tag{3}$$

and

$$Q_e = A \int_{T_R}^{T_e} (\lambda_e + \lambda_{Ri} + \lambda_{R*}) \, dT \tag{4}$$

Here $A = 2\pi L/\ln(R/r)$ is a form factor, T_R is the cold wall temperature (~ 700 K), γ and ε_ν are the absorption coefficient and emissivity of tungsten, and λ_a and λ_e are the atom and electron potassium plasma thermal conductivities. T_a and T_e are the temperatures of atoms and electrons near the filament, and $T_a = T_w - \delta T$, with δT the temperature jump. In our experiment, ionization and excitation processes can exist and are taken into account considering the reactive thermal conductivities λ_{Ri} and λ_{R*}.

If an axial magnetic field is applied, some of the electrons will stay longer in the volume between the hot wire and the cold wall. Between the collisions with heavy particles (ions and atoms), the electrons move along a trajectory determined by the cyclotron frequency $\omega = eB/m$. As a result, the perpendicular electron thermal conductivity $\lambda_{e\perp}$ and the related Q_e will decrease, while both U and T_w will increase. Thus, we obtain the basic equation of the method

$$Q(B) - Q(0) = I \times \Delta U = \frac{dQ_r}{dT_w} \Delta T_w + A\lambda_a \Delta T_a + A\left[\lambda_e \Delta T_e + \int_{T_R}^{T_e} (\lambda_{eB} - \lambda_e) \, dT_e\right] \tag{5}$$

where ΔU is the wire voltage change caused by the magnetic field B. In our experiment $\Delta U/U$ and $\Delta T_w/T_w$ were in the range 10^{-3} to 10^{-2} for $B = 0.364$ T, while the term $A\lambda_e \Delta T_e$ is much smaller than the other terms. The change in the heat transfer due to the ionization and excitation is negligibly small compared with the other terms.

3. DATA ANALYSIS

In the experiment we measured $\Delta U = U_B - U$ at fixed p_K and B for a given set of stable values of I. Thus, the change of the input electrical power $\Delta Q_B = I \Delta U$ is equal to the sum of the changes of the losses caused by the wire radiation (ΔQ_r), atom (ΔQ_a), and electron (ΔQ_e) thermal conductivities

$$\Delta Q_r + \Delta Q_a - I\,\Delta U = \Delta Q_e \tag{6}$$

The left side of Eq. (6) can be considered as experimentally determined $[\Delta Q_{exp} = I\,\Delta U - (\Delta Q_r + \Delta Q_a)]$, while the right side ($\Delta Q_{calc} = \Delta Q_e$) can be calculated using the known momentum transfer cross-section dependence on the electron energy $Q_{ea}(v)$. Then, $y = \ln(\Delta Q_{calc}/\Delta Q_{exp})$ will be a measure of the discrepancy between the theory and the experiments.

3.1. Magnetic Field Influence on the Wire, Atom, and Electron Temperatures

In a first approximation all three mechanisms of the heat transfer can be considered assuming that the increases in the atom ($\Delta T_a = T_{aB} - T_a$) and electron ($\Delta T_e = T_{eB} - T_e$) temperatures are equal to the increase in the wire temperature $\Delta T_w = T_{wB} - T_w$. ΔT_w (further denoted ΔT), which can be calculated

(a) from the measured V–A characteristics as

$$\Delta T = T_{wB}(R_{01}) - T_w(R_{02}) \tag{7}$$

($R_{01} = U_B/I$ and $R_{02} = U/I$ stand for the wire resistance measured in vacuum with and without magnetic field, respectively) or

(b) using the approximation formula

$$\Delta T \approx (dT_w/d\rho)(\Delta U_B/U)\,\rho(T_W) \tag{8}$$

where $\rho(T_W)$ is the tabulated tungsten resistivity [5]. The results of Eqs. (7) and (8) agree within 1 to 2%.

3.2. Magnetic Field Influence on the Heat Fluxes

3.2.1. The Hot-Wire Radiation Power Increment

The change of the radiation heat flux ΔQ_r is one of the main results of magnetizing the plasma. The accuracy of its calculation defines the reliability of the final data. Three alternative approaches were used to calculate ΔQ_r:

(a) By means of the Stephan–Boltzmann law.

$$\Delta Q_r = 2\pi\gamma\varepsilon_\nu({T_{wB}}^4 - {T_w}^4) \tag{9}$$

where γ and ε_ν are tabulated for tungsten in Ref. 5.

(b) From the V–A characteristics. We compared the points $R_1 = U_B/I$ and $R_2 = U/I$ measured in a potassium plasma at $I = \text{const}$ with the corresponding points of the vacuum characteristics with equal $R_{10} = U_{01}/I_{01}$ and $R_2 = U_{02}/I_{02}$. Then

$$\Delta Q_r = U_{01}I_{01} - U_{02}I_{02} \tag{10}$$

(c) Using the approximation formula.

$$\Delta Q_r = (dQ_r/dT_w)\,\Delta T \approx (dQ_r/d\rho)(\Delta U_B/U)\,\rho(T_w) \tag{11}$$

The derivatives (dQ_r/dT_w) and $\rho(dQ_r/d\rho)$ were calculated for our particular wire using the tabulated tungsten temperature dependences of the specific power $q_0(T_w)$ and resistivity $\rho(T_w)$ [5].

The comparison between the calculations made by means of Eqs. (9)–(11) showed acceptable agreement ($\approx 2\%$). The choice of $\varepsilon_\nu = 0.34$ to 0.43 in Eq. (9) resulted in a deviation of about $\pm 5\%$ with respect to the mean values. Therefore, we used only Eq. (9) as the simplest one.

3.2.2. Atom Conductivity Heat Flux

The magnetic field dependent increment of the atom conductivity heat flux

$$\Delta Q_a = A\left\{\int_{T_R}^{T_{aB}} \lambda_a\, dT - \int_{T_R}^{T_a} \lambda_a\, dT\right\} \cong A\lambda_a\,\Delta T \tag{12}$$

can be calculated using our previous low-temperature data on the monatomic potassium vapor thermal conductivity [3, 4],

$$\lambda_a(T) = \lambda_a(1100\text{ K}) \times (T_a/1100)^{0.91} \tag{13}$$

where $\lambda_a(1100\text{ K}) = 1.62 \times 10^{-2}\text{ W}\cdot\text{m}^{-1}\cdot\text{K}^{-1}$. The atom temperature is defined as $T_a = T_w - \delta T$. The temperature jump δT can be calculated as $\delta T = CQ_aT^{1/2}/p_K$ (C is a constant for a given geometry of the particular device). The calculated absolute values of ΔQ_a are approximately $0.1\Delta Q_r$.

3.2.3. Electron Conductivity Heat Flux

The magnetic field dependent decrement of the electron conductivity heat flux is a result of two contradictory effects: the increase in the electron temperature ($\Delta T_e = \Delta T_w$) and the decrease in the plasma electron thermal conductivity due to the magnetizing of the electrons [$\lambda_e(B) \angle \lambda_e(0)$]

$$\Delta Q_e = A\left\{\int_{1700}^{T_{eB}} \lambda_e(B)\, dT - \int_{1700}^{T_e} \lambda_e(0)\, dT\right\}$$

$$\cong \Delta Q_{\lambda e} + A\int_{1700}^{T_e} [\lambda_e(B) - \lambda_e(0)]\, dT \qquad (14)$$

where $\Delta Q_{\lambda e} = A\lambda_e(T_e)\,\Delta T$ is negligibly small compared to ΔQ_r ($\Delta Q_{\lambda e} \approx 0.01\Delta Q_r$). The calculation of the second term of Eq. (14) is based on the detailed theory of the transport properties of the partially ionized plasma developed in Ref. 6. The Coulomb interactions are taken into account in the calculation of the electron-heavy particles collision frequency following the Frost approximation [6, 7],

$$\nu^F = Nv Q_T(v) + a\frac{8\pi n}{T_e^{1/2}}(e^2/4\pi m\varepsilon_0)^2 (m/2k)^2 (\ln \Lambda - b/v^2) \qquad (15)$$

where N is the potassium atom density, n and m are the electron density and mass, ε_0 is the electrical permittivity of vacuum, k is the Boltzmann

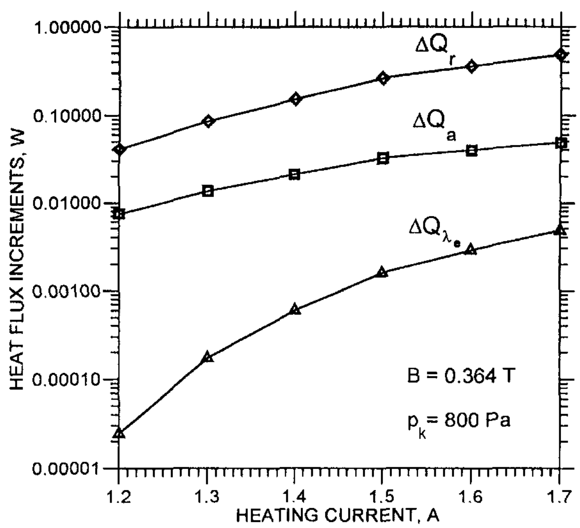

Fig. 1. Heat flux increments at constant p_k and B as functions of the wire heating current: (◇) for filament radiation heat losses; (□) for atom thermal conductivity heat losses, and (△) for electron thermal conductivity heat losses.

Table I. Relative Increments of the Heat Fluxes at $B = 0.364$ T and $p_k = 800$ Pa

	Filament heating current (A)					
	1.2	1.3	1.4	1.5	1.6	1.7
$\Delta Q_a/\Delta Q_r$	0.184	0.158	0.130	0.123	0.111	0.101
$\Delta Q_{\lambda e}/\Delta Q_r$	0.001	0.002	0.004	0.006	0.008	0.010

constant, $\ln \Lambda$ is the Coulomb logarithm, and a and b are constants different for the electrical conductivity σ, electron thermal conductivity λ_e, and thermal diffusion φ [7]. $Q_T = Q_{ea}(v) + Q_{exc}(v)$ is the total electron-heavy particle interaction cross section, with $Q_{exc}(v)$ being the inelastic interaction cross section.

The overheating of the plasma by the electric field ($E = U/L$) along the wire can be found from the equation,

$$\sigma E^2 = \frac{3mnk}{M}(\bar{\nu}_{ea} + \bar{\nu}_{ei})(T_e - T_a) \qquad (16)$$

where M is the potassium atomic mass, and $\bar{\nu}_{ea}$ and $\bar{\nu}_{ei}$ are the mean electron–atom and electron–ion collision frequencies, respectively.

The calculated increments of the heat fluxes ΔQ_r, ΔQ_a, and $\Delta Q_{\lambda e}$ as functions of the wire heating current I at $p_K = 800$ Pa and $B = 0.364$ T are presented in Fig. 1. The corresponding relative contributions $\Delta Q_a/\Delta Q_r$ and $\Delta Q_{\lambda e}/\Delta Q_r$ of the different mechanisms in the heat transfer are shown in Table I.

4. RESULTS AND DISCUSSION

Altogether $S = 82$ experimental points were measured and analyzed. The experimental conditions were carefully selected to meet the contradictory restrictions of the applied theoretical consideration. The influence of p_K, I, and B on the electron thermal conductivity in the hot-wire device is illustrated in Figs. 2–4.

In the calculation of T_e and ΔQ_e, we tried different $Q_{ea}(v)$: the semi-empirical cross section, derived from transport data [8], and the theoretical cross sections of Spencer and Phelps [9] and of Fabricant [10]. As expected, the best fit was achieved by a spline function minimizing only the deviations of our experiment. Unfortunately, such approximations based on a unique experiment are not universal. In the calculations of the transport coefficients we used the sum $Q_{ea}(v) + Q_{exc}(v)$, where Q_{exc} is the total

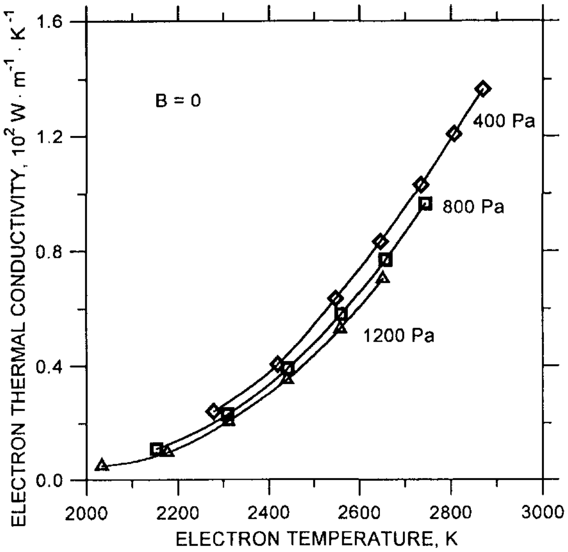

Fig. 2. Temperature dependence of the electron thermal conductivity in the absence of magnetic field at different p_K: (◇) 400 Pa, (□) 800 Pa, and (△) 1200 Pa.

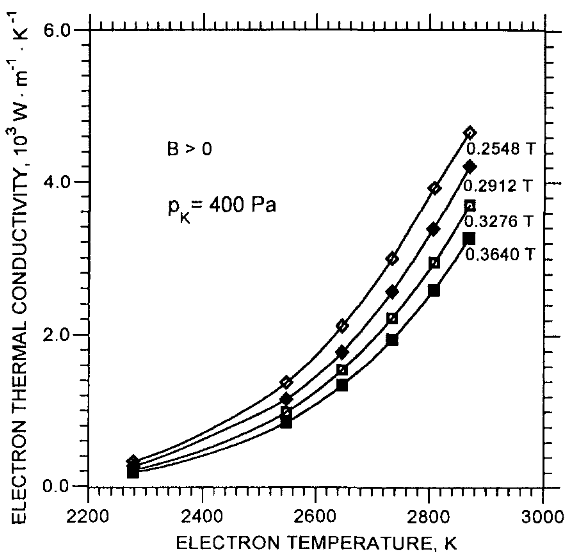

Fig. 3. Temperature dependence of the electron thermal conductivity at constant $p_K = 400$ Pa and different magnetic fields B: (◇) 0.2548 T, (◆) 0.2912 T; (□) 0.3276 T, and (■) 0.3640 T.

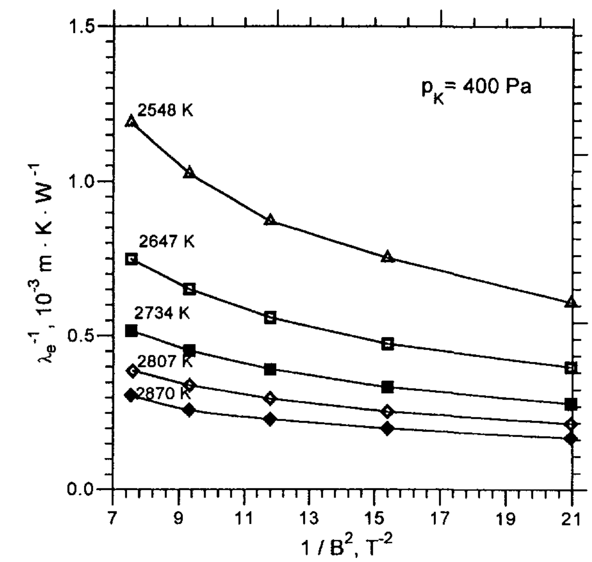

Fig. 4. Electron thermal conductivity as a function of magnetic field at constant $p_K = 400$ Pa and different T_e: (◆) 2870 K, (◇) 2807 K, (■) 2734 K, (□) 2647 K, and (△) 2548 K.

excitation cross section to the 4P level (1.6 eV) taken from the measurements of Chen and Galagher [11].

To get an idea about the order of the values considered in this work, we present in Table II an example of the calculated results at $B = 0.364$ T and $I = 1.5$ A for $p_K = 400$, 800, and 1200 Pa. All electron–atom cross section-dependent values (λ_e, T_e, ΔQ_e, $\Delta Q_{\lambda e}$, ΔQ_{calc}) were calculated with the electron velocity dependence of Q_{ea}, taken from Ref. 9. One more justification of the choice of the experimental conditions can be found in the fact that the best agreement between the theory and our experiment was obtained at $p_K = 800$ Pa ($y = 0.087$). At higher pressures ($p_K > 1200$ Pa),

Table II. Potassium Vapor Pressure Influence on the Heat Transfer [$B = 0.364$ T and $I = 1.5$ A, $Q_{ea}(v)$ from Ref. 9]

p_K (Pa)	T_w (K)	T_a (K)	T_e (K)	ΔT (K)	ΔQ (W)	ΔQ_r (W)	ΔQ_a (W)	ΔQ_{exp} (W)	ΔQ_{calc} (W)	$y = \ln(\Delta Q_{calc}/\Delta Q_{exp})$
1200	2300.5	2075.5	2352.0	2.72	0.048	0.151	0.020	0.123	0.080	0.185
800	2310.5	2001.6	2473.0	4.67	0.083	0.263	0.034	0.214	0.175	0.087
400	2315.0	1826.8	2679.0	8.91	0.158	0.506	0.060	0.408	0.516	−0.102

the breakdowns create discharge channels parallel to the filament. At lower pressures ($p_K < 400$ Pa), the accepted temperature jump approximation is not correct.

Comparing the deviations $y = \ln(\Delta Q_{calc}/\Delta Q_{exp})$ between the measured values and those calculated using one of the available electron–atom momentum transfer cross sections, we tried to evaluate their ability to predict transport properties of the partially magnetized potassium plasma. In his recent work Fabricant [10] found, by a comparative analysis of alkali vapors, that the theoretical momentum transfer cross sections are more reliable compared to the semiempirical ones, particularly for reproducing swarm data. Our comparison with the rms standard deviation seems to support this opinion, while the mean absolute deviation $\Sigma y/S$ is about three times less in the case of Ref. 8. The calculation of the electron thermal conductivity in a partially magnetized plasma is a complicated procedure. It includes calculations of the perpendicular and Hall components of the electrical conductivity and thermal diffusion. On the other hand, the estimated experimental error is relatively high (of the order of 30–40%). Because of these two inauspicious reasons we believe that both semiempirical and theoretical cross sections could be used to interpret our experimental results.

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